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Radiative heat transfer in insulations with random fibre orientation

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Abstract—The diffusion solution describes radiative flux in high temperature fibre insulations of sufficient optical thickness. This paper is concerned with the application of Mie-scattering theory for the determination of the extinction coefficient used in the diffusion equation. Emphasis is put on radiation incident oblique to the fibres. Averaging over all tilt angles reflects a completely random orientation of the fibres to the radiative flux. It is apparent that consideration of averaged oblique incidence is a realistic simulation of extinction efficiencies determined by guarded hot plate measurements.

1. INTRODUCTION

RADIATIVE heat flow has received attention from experimentalists and theorists for decades. In a low density fibre insulation infrared radiation is one of the most important heat transfer modes. In the case of reduced ambient pressure the heat conduction by the residual gas in the insulation decreases and for sufficiently low pressures radiative heat transfer becomes dominant even for low temperatures.

To improve the thermal performance of a fibre insulation the sensitivities of radiative transfer with respect to fibre characteristics have to be well understood. For a full understanding analysis is indispensible.

For a large optical thickness τ the diffusion solution

$$\dot{q}_{\rm rad} = \frac{16\bar{m}^2\sigma}{3\tau} T^3 \frac{dT}{dz}$$

is a valid approximation of the radiative transfer equations [1].

The optical thickness τ is determined from the anisotropic extinction coefficient E^* and the thickness L of the insulation. In the case of homogeneous E^* simply $\tau = E^*L$ holds true. To establish the dependence between extinction coefficients and fibre characteristics Mie-scattering theory is used.

Whereas scattering of electromagnetic radiation incident perpendicular to a cylinder has been considered in numerous publications, oblique incidence has received less attention. Infrared extinction coefficients $E^*_{\rm p}$ are computed for irradiance perpendicular to the fibres. Random oblique orientation is accounted for by multiplication of the computed extinction $E^*_{\rm p}$ by a constant factor $v = \pi/4 \simeq 0.785$ [8], $v = 2/3 \simeq 0.667$ [9] or $v = 4/5 = 0.800$ [10] thus yielding $E^*_{\rm o} = vE^*_{\rm p}$.

As indicated already by the different values of v the consideration of averaged oblique incidence by constant factors seems to be a first-order approximation valid for special cases. It will be demonstrated in this paper that the ratio between $E^*_{\rm p}$ and $E^*_{\rm o}$ varies with temperature and fibre diameter.

2. CALCULATION OF EXTINCTION FROM MIE THEORY

An electromagnetic wave incident on a fibre is scattered and absorbed. The relative scattering and absorption cross-sections $Q_{\rm sca}$ and $Q_{\rm abs}$ give the ratios between scattered and incident energy or absorbed and incident energy, respectively. The total extinction of the electromagnetic wave is described by the relative extinction cross-section $Q_{\rm ext} = Q_{\rm sca} + Q_{\rm abs}$.

The extinction cross-section can be calculated from Maxwell equations considering the real fibres as circular cylinders. Details may be found in the related literature [2–6]. One obtains for unpolarized infrared irradiation having a tilt angle ξ to the cylinder axis

$$Q_{\rm ext} = \frac{1}{x} Re\left\{a_0 + 2\sum_{n=1}^{\infty} a_n + b_0 + 2\sum_{n=1}^{\infty} b_n\right\}.$$

The terms a_n, b_n are dependent on Bessel and Hankel functions of the first kind

$$a_n = -\frac{A_nV_n - iC_nD_n}{W_nV_n + iD_n^2}$$

$$b_n = \frac{W_nB_n + iD_nC_n}{W_nV_n + iD_n^2}$$

where

NOMENCLATURE

D	fibre diameter	x	size parameter
E^*	anisotropic extinction coefficient	z	depth.
e^*	specific anisotropic extinction coefficient		
$H^{(1)}$	Hankel function of the first kind		
J	Bessel function of the first kind		
L	insulation thickness	Greek symbols	
m	index of refraction	λ	wavelength
p	scattering phase function	ζ	tilt angle
q_{rad}	heat flux due to radiation	ρ	density of insulation
Q	relative cross-sections	ρ_F	bulk density of fibres
T	temperature	σ	Stefan–Boltzmann constant
v	ratio of extinctions for oblique to perpendicular irradiance	τ	optical thickness
		ω	anisotropy factor for forward scattering.

$$A_n = \mathrm{i}\xi\{\xi J_n'(\eta)J_n(\xi) - \eta J_n(\eta)J_n'(\xi)\}$$

$$D_n = n\cos\zeta\,\eta J_n(\eta)H_n^{(1)}(\xi)\left(\frac{\xi^2}{\eta^2}-1\right)$$

$$B_n = \xi[m^2\xi J_n'(\eta)J_n(\xi) - \eta J_n(\eta)J_n'(\xi)]$$

$$C_n = n\cos\zeta\,\eta J_n(\eta)J_n(\xi)\left(\frac{\xi^2}{\eta^2}-1\right)$$

$$V_n = \xi[m^2\xi J_n'(\eta)H_n^{(1)}(\xi) - \eta J_n(\eta)H_n^{(1)\prime}(\xi)]$$

$$W_n = \mathrm{i}\xi[\eta J_n(\eta)H_n^{(1)\prime}(\xi) - \xi J_n'(\eta)H_n^{(1)}(\xi)]$$

$$\xi = x\sin\zeta;\quad \eta = x\sqrt{(m^2-\cos^2\zeta)}.$$

In the equations above m is the complex refractive index and $x = \pi D/\lambda$ where D is the diameter of the fibres and λ the wavelength of the irradiance.

For fibre insulations with sufficient density ρ the wavelength-dependent anisotropic extinction coefficient is given by

$$E_\lambda^* = \frac{\rho}{\rho_F}\frac{4}{\pi D}Q_{ext}(1-\omega)$$

where

$$\omega = \int_0^\pi \cos\varphi\, p(\varphi)\,\mathrm{d}\varphi \Big/ \int_0^\pi p(\varphi)\,\mathrm{d}\varphi$$

and $p(\varphi)$ the scattering phase function. Here ρ_F is the density of the fibres. The temperature dependent extinction coefficient $E^*(T)$ is obtained from the Rosseland mean of E_λ^*. The specific extinction e^* is defined by $e^* = E^*/\rho$.

3. APPLICATION TO RADIATIVE HEAT TRANSFER CALCULATION

The above equations are applied to the simulation of heat transfer measurements for an evacuated glass fibre insulation [8]. The complex index of refraction as given in ref. [7] is used. For the density of the fibres $\rho_F = 2700$ kg m^{-3} is considered [8]. The anisotropic specific extinction has been calculated for infrared radiation incident perpendicular ($\zeta = 0$) and oblique ($\zeta \geq 0$) to the fibres. To be more precise, the latter case accounts for all oblique orientations between 0 and $\pi/2$ and is therefore an average over all possible oblique incidence directions. The calculation results are presented in Figs. 1 and 2.

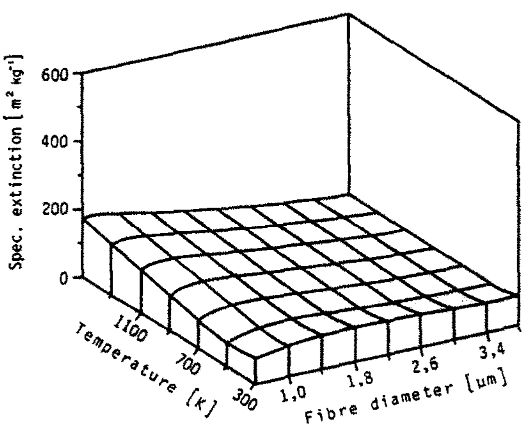

FIG. 1. Anisotropic specific extinction vs temperature and fibre diameter for perpendicular incidence ($\zeta = 0$).

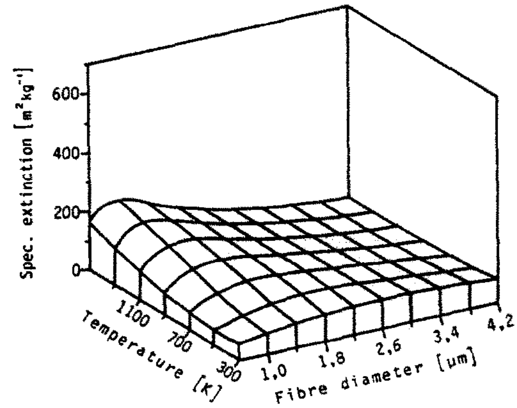

FIG. 2. Anisotropic specific extinction vs temperature and fibre diameter averaged over all oblique incidences ($0 \leq \zeta \leq \pi/2$).

Table 1. Specific extinction for perpendicular (e_p^*) and oblique (e_o^*) incidence (averaged over all incidence angles)

T [K]	e_p^* [m^2 kg^{-1}]			e_o^* [m^2 kg^{-1}]		
300.0	75.9	83.9	90.9	56.1	62.7	69.1
400.0	68.0	78.9	87.1	51.1	61.5	71.2
500.0	67.7	81.2	89.8	50.5	64.9	77.7
600.0	72.4	88.1	96.4	53.6	72.3	88.4
700.0	80.1	97.5	104.0	59.5	82.8	102.0
Fibre diameter [μm]	0.6	0.8	1.0	0.6	0.8	1.0

Table 2. Ratio between specific extinctions due to perpendicular and averaged oblique radiation incidence

T [K]	$v = e_o^*/e_p^*$		
300	0.739	0.747	0.760
400	0.751	0.779	0.817
500	0.746	0.799	0.865
600	0.740	0.821	0.917
700	0.749	0.849	0.981
Fibre diameter [μm]	0.6	0.8	1.0

Table 3. Comparison between the present analysis ($D = 0.6$ μm) and previous [8] results ($D = 0.5$–0.7 μm)

	Present analysis	Theoretical results [8]	Experimental results [8]
$\bar{e}_p^*$	70.5	60–76	—
$\bar{e}_o^*$	52.6	47–60	—
$\bar{e}_{exp}^*$ [m^2 kg^{-1}]	—	—	53
$\bar{v}$	0.745	0.785	—

It is noted that the maxima of the surface $e^*(D, T)$ are more pronounced for oblique than for perpendicular incidence.

Some numerical results are shown in Table 1. Table 2 presents the ratio between specific extinction due to oblique (e_o^*) and perpendicular (e_p^*) incidence, i.e. $v = e_o^*/e_p^*$. Obviously this ratio changes with temperature and fibre diameter.

The extinction of the evacuated glass fibre insulation has been measured [8] in the radiation mean temperature range 300 K to nearly 500 K. Properties of the tested insulation are [8]: insulation thickness, 30 mm; fibre diameter, 0.5–0.7 μm; insulation density $\rho = 280$ kg m^{-3}. A temperature averaged specific extinction $e_{exp}^* = 53$ m^2 kg^{-1} was determined for that insulation by guarded hot plate experiments [8].

From Mie theory (perpendicular incidence) temperature averaged extinctions $e_p^* = 60$ and 76 m^2 kg^{-1} have been calculated for the fibre diameters of 0.5 and 0.7 μm [8]. Oblique incidence was accounted for by multiplication of all calculated extinctions with

$$\int_0^{\pi/2} \cos^2 \varphi \, d\varphi = \pi/4 \simeq 0.785$$

thus yielding $e_o^* = 47$ m^2 kg^{-1} ($D = 0.5$ μm) and $e_o^* = 60$ m^2 kg^{-1} ($D = 0.7$ μm) [8].

Temperature averaging our results presented in Table 1 for a (mean) fibre diameter $D = 0.6$ μm in the temperature range 300–500 K one obtains the results compared in Table 3 with the values given in ref. [8].

4. CONCLUSIONS

The agreement between the measured extinction $e_{exp}^* = 53$ m^2 kg^{-1} and the present calculations is excellent for averaged oblique incidence $e_o^* = 52.6$ m^2 kg^{-1} (Table 3).

Consideration of averaged oblique incidence by a constant factor 0.785 [8] fits the extinction of this special measurement too and the factor is quite close to our calculated mean value $e_o^*/e_p^* = 0.745$ (Table 3). However, Table 2 indicates in the general case no constant relation between extinctions for perpendicular and averaged oblique incidence.

REFERENCES

1. R. Siegel and J. R. Howell, *Thermal Radiation Heat Transfer*. McGraw-Hill, New York (1972).
2. H. C. Van de Hulst, *Light Scattering by Small Particles*. Wiley, New York (1957).
3. C. F. Bohren and D. R. Huffman, *Absorption and Scattering of Light by Small Particles*. Wiley, New York (1983).
4. B. K. Larkin and S. W. Churchill, Scattering and absorption of electromagnetic radiation by infinite cylinders, *J. Opt. Soc. Am.* **49**(2), 188–190 (1959).
5. A. C. Lind and J. M. Greenberg, Electromagnetic scattering by obliquely oriented cylinders, *J. Appl. Phys.* **37**(4), 3195–3202 (1965).
6. M. Kerker, D. Cooke, W. A. Farone and R. A. Jacobsen, Electromagnetic scattering from an infinite circular cylinder at oblique incidence, *J. Opt. Soc. Am.* **56**(4), 487–491 (1966).
7. C. K. Hsieh and K. C. Su, Thermal radiative properties of glass from 0.32 to 206 μm, *Solar Energy* **22**, 37–43 (1979).
8. R. Caps, A. Trunzer, D. Büttner, J. Fricke and H. Reiss, Spectral transmission and reflection properties of high temperature insulation materials, *Int. J. Heat Mass Transfer* **27**, 1865–1872 (1984).
9. *VDI-Wärmeatlas*. VDI-Verlag GmbH, Düsseldorf (1988).
10. R. Caps, Strahlungswärmeströme in evakuierten thermischen Superisolationen, Diss., Phys. Inst. Univ. Würzburg (1985).

TRANSFERT RADIATIF DANS DES ISOLANTS A ORIENTATION ALEATOIRE DES FIBRES

Résumé—La solution de diffusion décrit le flux radiatif à haute temperature dans des isolants fibreux d'épaisseur optique suffisante. Ce texte concerne l'application de la théorie de Mie pour la détermination du coefficient d'extinction utilisé dans l'équation de diffusion. L'intérêt est porté sur le rayonnement incident oblique par rapport aux fibres. Une moyenne pour tous les angles d'inclinaison représente une orientation complètement aléatoire des fibres pour le flux radiatif. Il apparait que la considération d'une incidence oblique moyenne est une simulation réaliste des efficacités d'extinction déterminées par des mesures de plaque chaude gardée.

STRAHLUNGSWÄRMEUSTAUSCH IN WÄRMEDÄMMUNGEN MIT ZUFÄLLIGER FASERORIENTIERUNG

Zusammenfassung—Es wird der Strahlungswärmeuastausch in faserigen Wärmedämmaterialien für den Hochtemperaturbereich bei ausreichender optischer Dicke beschrieben. Die Streutheorie nach Mie wird bei der Bestimmung des Extinktionskoeffizienten in der Diffusionsgleichung Angewandt. Besonderer Wert wird auf die schräg in die Faser einfallende Strahlung gelegt. Eine Mittelung über alle Einfallswinkel spiegelt eine wirklich zufällige Anordnung der Fasern zum Strahlungsfluß wider. Es ist offensichtlich, daß die Betrachtung geneigter schräger Einstrahlungen eine realistische Simulation der Extinktionswirkungsgrade darstellt, wie sie aus Heißplattenmessungen bekannt sind.

РАДИАЦИОННЫЙ ТЕПЛОПЕРЕНОС В ИЗОЛЯЦИЯХ С ПРОИЗВОЛЬНОЙ ОРИЕНТАЦИЕЙ ВОЛОКОН

Аннотация—При помощи решения уравнения диффузии описывается лучистый поток в высокотемпературных волокнистых изоляциях с достаточной оптической толщиной. Рассматривается применение теории рассеяния Ми для определения коэффициента ослабления, используемого в уравнениях диффузии. Особое внимание уделяется излучению, наклонно падающему на волокна. Усреднение по всем углам наклона отражает совершенно произвольную ориентацию волокон относительно лучистого потока. Очевидно, что исследование усредненного наклонного падения излучения проводится на основе реалистического моделирования эффективностей ослабления, определяемых измерением с использованием нагретых охранных пластин.